

MITIGATION OF EMITTANCE DILUTION DUE TO TRANSVERSE MODE COUPLING IN THE L-BAND LINACS OF THE ILC[†]

R.M. Jones[‡], and R.H. Miller, SLAC, Stanford, CA 94309, USA

Abstract

The main L-band linacs of the ILC accelerate 2820 bunches from a center of mass of 10 GeV to 500 GeV (and in the proposed later upgrade, to 1 TeV). The emittance of the vertical plane is approximately 400 times less than that of the horizontal plane. Provided the vertical and horizontal mode dipole frequencies are degenerate then the motion in each plane is not coupled. However, in reality the frequency degeneracy is split and the eigenmodes are shifted –due to inevitable manufacturing errors introduced in fabricating 20,000 cavities. This gives rise to a transverse coupling in the horizontal-vertical motion and can readily lead to a dilution in the emittance in the vertical plane. We investigate means to ameliorate this effect dilution by splitting the horizontal-vertical tune of the lattice.

INTRODUCTION

In a linear collider it is important to maximize the luminosity of the colliding beams at the interaction point. It can be shown [1] that the luminosity is inversely proportional to the square root of the product of the vertical and horizontal beam emittances. The emittance is driven by wakefields excited by charged bunches traversing each of main cavities in the ILC [2]. Initial studies [3] have indicated that cross-coupling of modes may lead to a significant dilution of the vertical emittance. We investigate the effect of changing the horizontal to vertical phase advance of the linac's lattice on the vertical emittance of the beam to mitigate the emittance dilution.

The next main section discusses the wakefields associated with the mode frequency splitting and the penultimate main section deals with simulations of the effect of these wakefields on the final beam emittance.

TRANSVERSE WAKEFIELDS

Fabrication errors will result in cavities having imperfect axial symmetry of the cavities and this results in two resonance frequencies [4] of the cavity modes from what would be a design value of a single degenerate frequency. The overall distortion in the cavity shape and the relative position of the HOM couplers with respect to the cavities results in a rotation of the eigenmodes from their horizontal-vertical orientation. We consider the deflecting mode of the cavity being distorted by rotating it through an angle, ϕ . Thus, as the beam travels down the linac the electromagnetic field rotates and the horizontal kick to the beam is coupled to the vertical motion of the beam. The initial voltage kick along the rotated coordinate frame $x'-y'$ (illustrated in fig. 1) is given by:

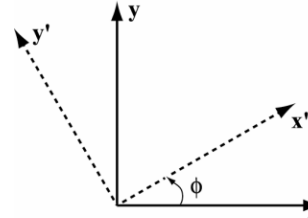


Figure 1: Mode axes for rotational transformation.

$$\left. \begin{aligned} V_{x'}(0) &= V_x(0) \cos \phi \\ V_{y'}(0) &= -V_x(0) \sin \phi \end{aligned} \right\} \quad (1.1)$$

The resulting voltage kick at any instant in time is then:

$$V_y(t) = V_{x'}(t) \sin \phi + V_{y'}(t) \cos \phi \quad (1.2)$$

Substituting (1.2) into (1.1) and assuming a time variation of the form $V_{x'}(t) = V_{x'}(0) \exp(j\omega_{x'} t)$, enables the vertical kick that the beam receives to be obtained in terms of the x motion:

$$V_y(t) = jV_x(0) \sin 2\phi \exp(j\bar{\omega} t) \sin(\Delta\omega t/2) \quad (1.3)$$

where the average frequency is given by $\bar{\omega}$ and the frequency degeneracy splitting is given by: $\Delta\omega = \omega_{x'} - \omega_{y'}$. The cross-coupled transverse wakefield corresponding to this dipole kick is given by:

$$W_t^{xy}(t) = \sum_p K_p^x \cos(\bar{\omega}_p t) \sin(\Delta\omega_p t/2) \exp(-\bar{\omega}_p t/2Q_p^x) \quad (1.4)$$

where the p^{th} modal kick factor and quality factor are given by K_p^x and Q_p^x , respectively. In addition to this cross-coupled term, there is the usual vertical wake [5]:

$$W_t^{yy}(t) = \sum_p K_p^y \sin(\omega_p^y t) \exp(-\omega_p^y t/2Q_p^y) \quad (1.5)$$

where the y superscript indicates a vertical (non-cross-coupled) quantity. This component of the wake is also expected to rotate, but this effect is not treated here. The wakefield that the beam experiences as it travels down the linac will be a combination of (1.4) and (1.5) and will not, of course, be identical for each cavity. In practice random errors introduced during the process of fabrication different kicks to the beam from each cavity.

BEAM DYNAMICS

Well-Damped Modes

We track the beam down the linac using the code LIAR [6] for various horizontal to vertical tunes of the FODO lattice. The results of these detailed simulations are illustrated in fig 2 and fig 3. In all simulations the beam is subjected to randomly distributed wakefields along the entire linac. Also, the beam is injected at 5 GeV, with no vertical offset, a horizontal offset of 400 μm ($\sim \sigma_x$) and exits the linac at 250 GeV. The modes which constitute these wakes are all sufficiently well-damped such that little emittance dilution occurs when there is no mode

[†] Supported by the U.S. DOE grant number DE-AC03-76SF00515

[‡] Now at University of Manchester, Dept of Physics and Cockcroft Institute, Cheshire, UK

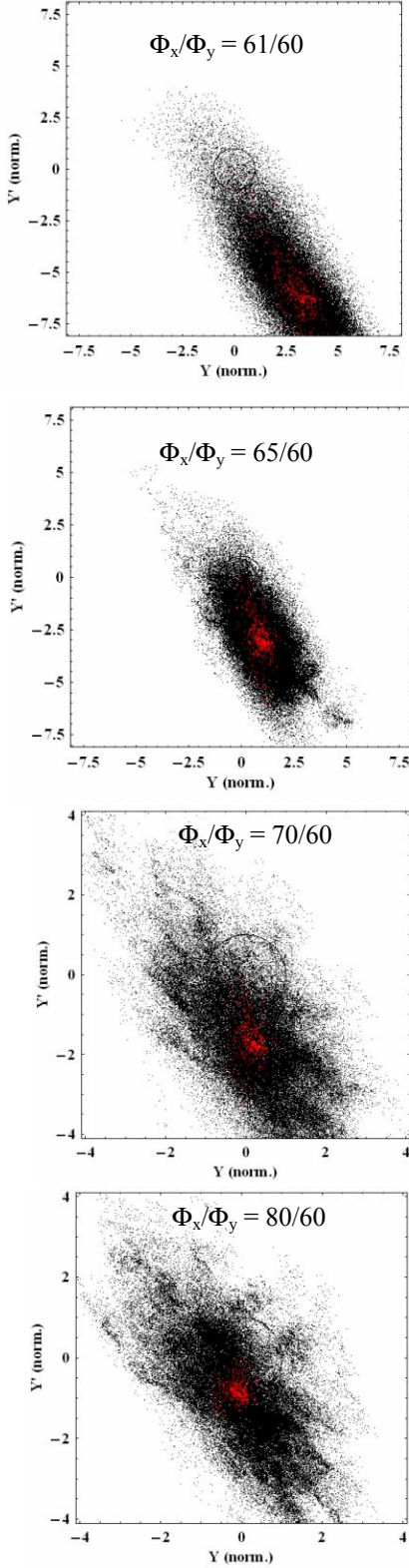


Figure 2: Phase space at end of linac. The results of tracking 500 bunches down 200 machines under the influence of long-range wakes are illustrated in (b) through (d) for several different lattice tune ratios. The transverse dimensions of the beam are: $\sigma_{x,y} \sim 10 \mu\text{m}, 270 \mu\text{m}$. The red dots correspond to averaging over all 200 linacs.

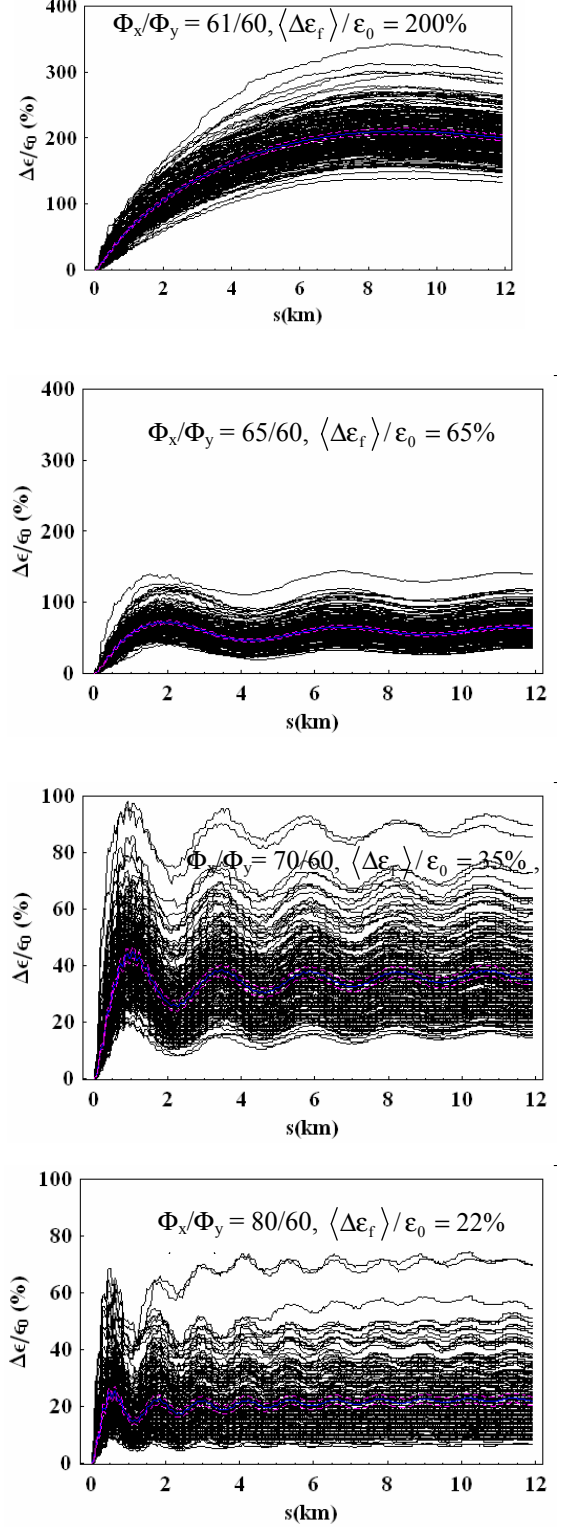


Figure 3: Emittance dilution down linac for the corresponding 200 machines in fig 2. The mean emittance dilution for all machines is indicated by the solid blue line (the final mean emittance dilution, $\langle \Delta \epsilon_f \rangle / \epsilon_0$ is also indicated on each figure). Initial parameters: $\epsilon_{x,y} = 8000, 20 \text{ nm.rads}$, $\beta_{x,y} \sim 89, 51 \text{ m}$, $\sigma_{x,y} (= \epsilon_{x,y} \beta_{x,y} / \gamma)^{1/2} \sim 10, 270 \mu\text{m}$

coupling [1]. Furthermore, as there is no vertical offset then any emittance dilution that occurs must accrue entirely from coupling the horizontal to vertical motion of the beam as it traverses the linac. In all simulations presented, the effects of long-range wakefields on the beam when the couplers are at a fixed azimuthal distribution along the linac are considered. The first case shown in figs 2(a) and 3(a) the tune of both the vertical and horizontal planes differs by no more than one degree and this results in a mean emittance dilution in the vertical plane of $\sim 200\%$, with a worst case machine emittance dilution of 350% . Changing the horizontal to vertical phase advance of the lattice to 65:60 is illustrated in fig 2(b) and fig 3(b) and this considerably reduces the mean emittance dilution to 65% . Further increases in the tune ratio, are illustrated in fig 2(c,d) and fig 3(c,d). Splitting the phase advance of the lattice by at least 10 degrees does not allow the transverse coupling to occur resonantly and reduces the mean emittance dilution to manageable levels. However, the worst case machine still displays a significant emittance dilution at the end of the linac. For example, for a ratio of horizontal to vertical phase advance of 70 to 60 the emittance dilution is almost 100% . The emittance dilution can be further controlled by improving the damping of the modes which constitute the wakefield by reducing their characteristic Q values. Further work is required in this area on ascertaining the required damping which brings down the emittance dilution to acceptable levels. Finally, we note that in practice the couplers will be randomly distributed azimuthally along the linac. Simulations (not shown here) indicate that the qualitative behaviour is not changed at all by adding random azimuthal orientations of the couplers by 10% about their mean value. The actual azimuthal spread of the couplers about their design value is not known at this stage.

Single-Badly Damped Mode

We consider the same dipole mode spectrum as utilized in the previous section except that a high frequency mode is poorly damped. The Q at 2.575 GHz has been increased by a factor of 10 to 5×10^5 . On including random frequency errors, we calculate 50 different wakefields and

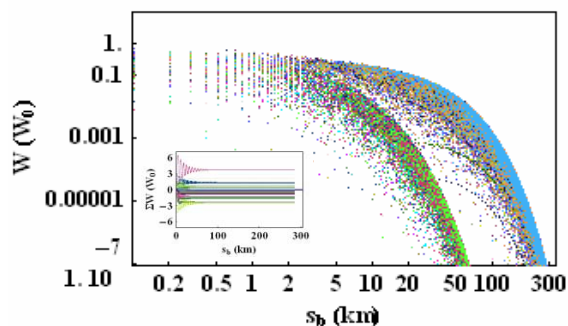


Figure 4: Fifty separate wakefields (represented by a separate color) at the location of each bunch. The wake is normalized with respect to $W_0=0.11$ V/pC/mm/m. Shown inset, is the sum wakefield for each of the 50 wakefields.

these are illustrated in fig. 4, together with the sum wakefield. The sum wakefield at a particular bunch is defined as the sum of the wakes at all previous bunches [7]. There are resonances in the sum wakefield in which it is significantly larger than unity. From past experience with the simulation of the beam dynamics of X-band accelerating structures we expect BBU [8] to occur when the wake is appreciably larger than unity. Monitoring the emittance dilution as the beam is tracked down the complete linac with the code LIAR is illustrated in fig 5.

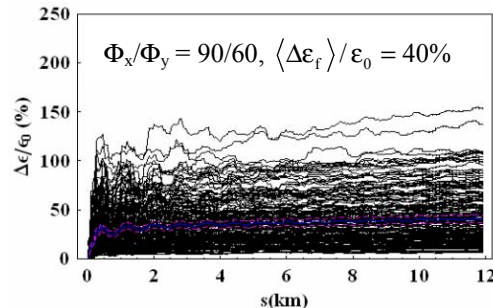


Figure 5: Emittance dilution down linac for 200 different machines under the influence of fixed azimuthal location of HOM couplers. All wakes have one mode which is poorly damped. The blue line corresponds to the average emittance dilution.

In the simulation, 50 wakefields have been distributed randomly throughout the entire linac, the azimuthal distribution of couplers have been fixed such that $\phi=\pi/4$ and the horizontal to vertical phase is in the ratio 90 to 60 degrees. In this situation the final emittance dilution, averaged over 200 machines is 40% . The maximum dilution of one particular machine in the simulation is 150% . This is compared with the case $\Phi_x/\Phi_y=1$ which results in a 648% mean dilution in the emittance. Thus, splitting the tune of the lattice reduces the emittance dilution significantly.

CONCLUSIONS

Further detailed simulations into the effect of mode rotation on the beam emittance are required. However, these initial simulations suggest that the emittance dilution that accrues due to mode coupling may be reduced considerably by splitting the tune of the lattice.

ACKNOWLEDGEMENTS

This work has benefited from stimulating discussions in the weekly ILC structures meetings at SLAC. We are grateful to Peter Tenenbaum for modifying LIAR.

REFERENCES

- [1] N. Baboi *et al.*, LINAC04, SLAC-PUB-10684
- [2] <http://www.interactions.org/linearcollider/>
- [3] R.M. Jones *et al.*, PAC05, SLAC-PUB-11234
- [4] M. Ross *et al.*, PAC05, SLAC-PUB-11190
- [5] P.B. Wilson, SLAC-PUB-4547
- [6] R. Assman *et al.*, LIAR, SLAC-PUB-AP-103
- [7] R.M. Jones *et al.*, LINAC04, SLAC-PUB-10683
- [8] K. Yokoya, DESY Report 86-084, 1986